Technical Note: Conditions for Similarity of Mass-Transfer Coefficients and Fluid Shear Stresses Between the Rotating Cylinder Electrode and Pipe

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ABSTRACT

A debate surrounds the question of whether to use equality of the mass-transfer coefficient or of the fluid shear stress at the wall to define flow parameters to simulate velocity-sensitive corrosion between geometries. In principle, this debate may be circumvented if both parameters could be simultaneously similar in the modeled and modeling geometries. To explore this possibility, an equation derived from the analogy among heat, mass, and momentum transfer was used to estimate rotating cylinder electrode diameters and rotation rates that enable simultaneous similarity of the mass-transfer coefficient and the shear stress at the wall between hydraulically smooth cylinders and straight pipes. The results imply that under some but not all conditions, practical cylinder diameters and rotation rates may be found.

KEY WORDS: fluid shear stress, mass-transfer coefficient, pipe, rotating cylinder electrode, velocity

INTRODUCTION

Developing appropriate protocols for using laboratory equipment such as the rotating cylinder electrode (RCE) to aid in predicting if, how, and to what degree fluid flow affects corrosion in field environments is an active area of research. That a measurement in one geometrical configuration can be used to predict velocity-sensitive corrosion in another is an assumption. One argument in its favor is as follows. The large Schmidt numbers normally encountered in liquids mean that the fully developed mass-transfer boundary layer for hydraulically smooth surfaces is much thinner than the fully developed hydrodynamic boundary layer. This relationship should exist in all situations in which mass and momentum transfer are governed by the presence of fully developed boundary layers. Under turbulent flow conditions most of the change in velocity between the wall and free stream occurs over a fairly narrow distance from the wall. In a smooth pipe, for example, approximately 80% of the change occurs over the first 10% of distance into the fluid.[1] This observation means that much of the transfer of momentum between the fluid and the wall occurs in this limited region. The mass-transfer boundary layer thickness in a liquid of Schmidt number 1,000 (approximately that of water) is about an order of magnitude smaller. This relationship suggests that the mass-transfer boundary layer thickness could be much smaller than the radius of curvature. The implication is that the coordinate system within the fully developed mass-transfer boundary layer may in a practical sense be independent of the geometric shape.

The objective, then, is to define experimental conditions in one type of geometry, which can enable that geometry to model and predict the effect of fluid motion on corrosion in a different geometry. Note that the word "predict" does not necessarily mean an exact one-for-one quantitative prediction of the corrosion rate. The word "predict" does mean that the corrosion mechanism should be the same in the two geometrical configurations.

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Requiring similarity in wall shear stress[2-3] and similarity in mass-transfer coefficients[4-5] have both been proposed to establish flow conditions (rpm) within the RCE that could enable predictions of velocity-sensitive corrosion mechanisms in other geometries. Which approach is most appropriate is a subject of debate.[2,4] As suggested earlier,[2] for fully developed turbulent flow, the two approaches are linked because of the relationship that exists between the friction factor and the mass-transfer coefficient. Calculations have suggested that, at least under some circumstances, ensuring the equality of mass-transfer coefficients results in a similarity in wall shear stresses[6] between geometries. One possible way to make corrosion measurements that satisfy such a similarity in both criteria simultaneously would be to use experimental conditions under which the wall shear stresses and the mass-transfer coefficient are both simultaneously similar between the field and experimental configurations. An approximate method for estimating such conditions is as follows.

The Stanton number written for mass transfer relates the flux of mass entering the fluid from the surface by mass transfer to the total flux of material passing by the surface because of fluid flow. The relationship can be written as:

$$St = \left(\frac{Sh}{Re\,Sc}\right) = \frac{k}{u} = \left(\frac{f}{2}\right)Sc^{c-1} \quad (1)$$

The wall shear stress is related to the friction factor by:

$$\tau = (f/2)\rho u^2 \quad (2)$$

Then substituting Equation (2) into Equation (1) results in:

$$\tau = \rho k u Sc^{1-c} \quad (3)$$

Following is how this method might be applied to using the RCE to model corrosion in a straight pipe. Equation (3) suggests that if conditions are established in the same environment so that both the mass-transfer coefficients and appropriate fluid velocities are set equal simultaneously in the two geometries, then the shear stress at the wall would also be equal in the two geometries. The exponent, 1-c, on the Schmidt number for the rotating cylinder (0.644) has been assumed to be equal to that for the pipe (0.667). This concept means that if the RCE could be operated so that both its mass-transfer coefficient and peripheral velocity[7] are the same as the mass-transfer coefficient and mean velocity (volumetric flow rate divided by area) in the pipe,[8] the fluid shear stresses at the wall would be similar. Further, the mass-transfer-affected corrosion mechanism might be modeled independent of whether equality of wall shear stress or equality of mass-transfer coefficient is the most appropriate approach.

The value of 1 has been assumed for the exponent on the friction factor for both geometries. This assumption is important to note. The Chilton-Colburn analogy[9] among heat, mass, and momentum transfer suggests a value of 1. But this analogy, which was originally derived for gases, has been indicated to overestimate the Stanton number in pipes for liquids by 20% to 30%.[10] Churchill has suggested an exponent of 0.5.[11] Hubbard and Lightfoot[12] and Berger and Hau[10] have suggested that the exponent lies between 0.5 and 1. With respect to the RCE, Eisenberg, et al.,[13] and Singh and Mishra[14] have concluded that the exponent on the friction factor in Equation (1) should be 1. To enable the derivation of a simple, analytical equation, the exponent on the friction factor was assumed to be 1 in this analysis for both the pipe and RCE. The result is that the results shown later are only approximate. That approximation does not detract from the overall conclusion that practical conditions may be estimated to make the mass-transfer coefficient and fluid shear stress at the wall simultaneously similar in both geometries.

The pipe and rotating cylinder were assumed to be hydraulically smooth. Previously, equations were developed to estimate the relationship between the velocities in the RCE and pipe that would create equal mass-transfer coefficients.[4,15] These equations were derived using the log-linear correlation of the Sherwood number (mass-transfer coefficient) vs. Reynolds number data in these two geometries. Equation (4) taken from an earlier work[4] is one example:

$$u_{cyl} = 0.1185\left[\left(\frac{\mu}{\rho}\right)^{-0.25}\left(\frac{d_{cyl}^{3/7}}{d_{pipe}^{5/28}}\right)Sc^{-0.0857}\right]u_{pipe}^{5/4} \quad (4)$$

If u_{cyl}, the peripheral cylinder velocity, is set equal to u_{pipe}, the mean pipe fluid velocity, both the velocities and mass-transfer coefficients in the two geometries are equal. Equations (1) through (3) would then suggest that the shear stresses at the wall would be simultaneously similar. By setting u_{cyl} and u_{pipe} equal to each other, Equation (4) can be rearranged so that the RCE diameter can be estimated as a function of the pipe diameter. This rearrangement results in:

$$d_{cyl} = \left[8.442 d_{pipe}^{0.1786} Sc^{0.0857}\left(\frac{\mu}{\rho}\right)^{0.25} u^{-0.25}\right]^{2.333} \quad (5)$$

Equation (5) provides a straightforward approach to estimating the cylinder diameter required to enable simultaneous similarity of both wall shear stresses and mass-transfer coefficients between the pipe and RCE. The subscript on "u" has been eliminated because it would be set equal in the two geometries.

Equation (5) can be used to estimate the required diameter of the rotating cylinder as a function of pipe velocity when the diameter of the pipe is known. Figure 1 shows the cylinder diameters as functions of pipe velocity for pipe diameters of 5 cm, 10 cm, 20 cm, 50 cm, and 100 cm. Figure 2 shows the corresponding rotation rates (in rpm). The viscosity, density, and Schmidt numbers were assumed to be 0.01 centipoise, 1 g/cm^3, and 1,000, respectively. Two implicit boundaries are the Reynolds number limits of the correlations used in Equations (4) and (5), from less than 1,000 to greater than 10^5 for the cylinder and 2,300 to greater than 10^6 for the pipe. The lines in the figures have been drawn so that all pipe Reynolds numbers would be turbulent. All RCE Reynolds numbers were turbulent. An alternative to Equation (4) has been proposed[15] to expand the upper limits on the Reynolds number for both geometries. A relationship equivalent to Equation (5) derived from that newer equation was also examined, but the results were found to be very similar to those in Figures 1 and 2.

Not all of the dimensions and rotation rates suggested in Figure 1 can be achieved in the laboratory. Limitations may exist on the electrode diameter because of the configuration of the rotating cylinder apparatus and on rotation rates because of the drive mechanism. The dotted lines with arrows in both figures have been drawn to show what range of values of cylinder diameter, pipe diameter, and pipe velocity would enable cylinder rotation rates between 10 rpm and 10^4 rpm, and cylinder diameters between 1 cm and 10 cm.

The results imply that some combinations of pipe diameter, pipe velocity, rotating cylinder diameter, and rotation rate exist so that the mass-transfer coefficients and the wall shear stresses can be similar simultaneously. Fulfilling both criteria where possible might circumvent the need to consider whether wall shear stress or mass-transfer coefficient is the appropriate modeling parameter. Note that experimental confirmation is needed to show that the conditions implied do, indeed, result in the simultaneous similarity of both fluid shear stresses at the wall and mass-transfer coefficients in the two geometries. Such verification would require mass-transfer coefficients and wall shear stresses to be measured in both geometry types in the same environment under conditions that are proposed to fulfill Equation (5).

In addition, note that the word "similarity" has been used in place of "equality" with respect to the relationships between geometries for two reasons. First, Equation (5) was derived assuming that the exponent on the friction factor is 1 in the two geometries. That assumption should result in an overestimate of the shear stress at the wall in the straight pipe. Second, Equation (4) was developed using Sherwood number vs. Reynolds number relationships that are derived from log-linear correlations of measured Sherwood numbers or friction factors as a function of the Reynolds number. As discussed previously, the actual relationship is curved when plotted as a function of the Reynolds number.[6] The log-linear approximation of the curvature seems to create an uncertainty factor of 2 to 3 between calculated and actual Sherwood numbers, mass-transfer coefficients, and, by inference, friction factors. Results derived from equations such as Equations (4) and (5) above should be viewed accordingly.

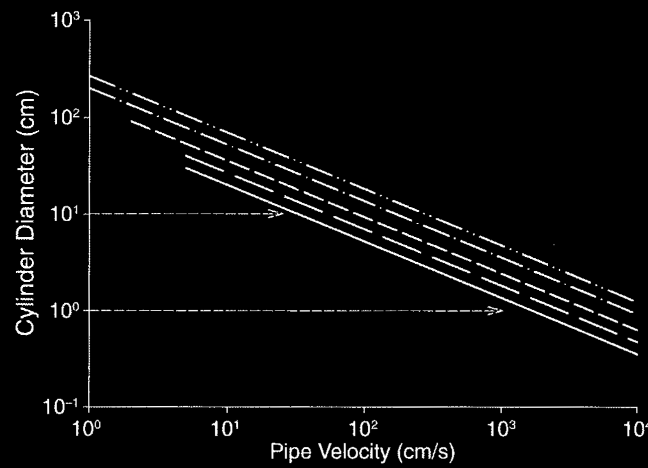

FIGURE 1. *Cylinder diameter as function of pipe velocity to fulfill Equation (4). The region of experimentally practical cylinder diameters lies between the two arrows. Pipe diameters = (———) 5 cm, (— — —) 10 cm, (– – –) 20 cm, (– · – · ·) 50 cm, and (– ·· – ·· – ··) 100 cm.*

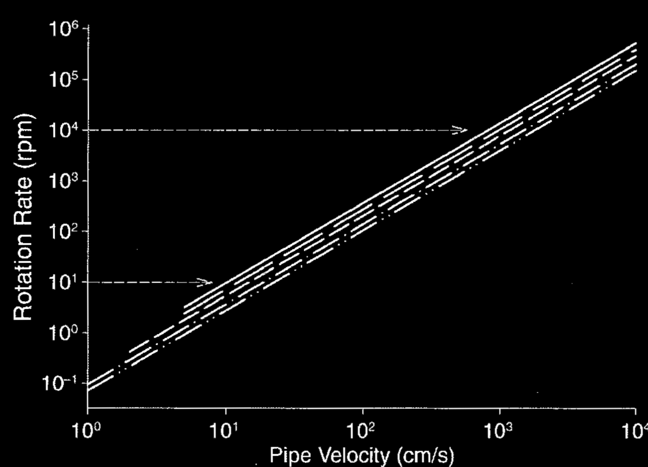

FIGURE 2. *Cylinder rotation rates as function of pipe velocity to fulfill Equation (4). The region of experimentally practical rotation rates lies between the two arrows. Pipe diameters = (———) 5 cm, (— — —) 10 cm, (– – –) 20 cm, (– · – · ·) 50 cm, and (– ·· – ·· – ··) 100 cm.*

LIST OF SYMBOLS

d diameter (cm)

D diffusion coefficient in solution (cm^2-s^{-1})

f friction factor (dimensionless)
k mass-transfer coefficient (cm-s^{-1})
Re Reynolds number (dimensionless) = $\left[\frac{\rho(\varpi d)d}{\mu}\right] = \left(\frac{\rho u d}{\mu}\right)$
Sc Schmidt number (dimensionless) = $\left(\frac{\mu}{\rho D}\right)$
Sh Sherwood number (dimensionless) = $\left(\frac{kd}{D}\right)$
St Stanton Number (dimensionless) = $\left[\frac{(Sh)}{(Re)(Sc)}\right]$
u fluid velocity (cm-s^{-1})
μ absolute viscosity (g-cm^{-1}-s^{-1})
ρ density (g-cm^{-3})
τ fluid shear stress at surface or wall (dyne/cm^2)
ω rotation rate (radians-s^{-1})

Subscripts

cyl rotating cylinder electrode
pipe pipe

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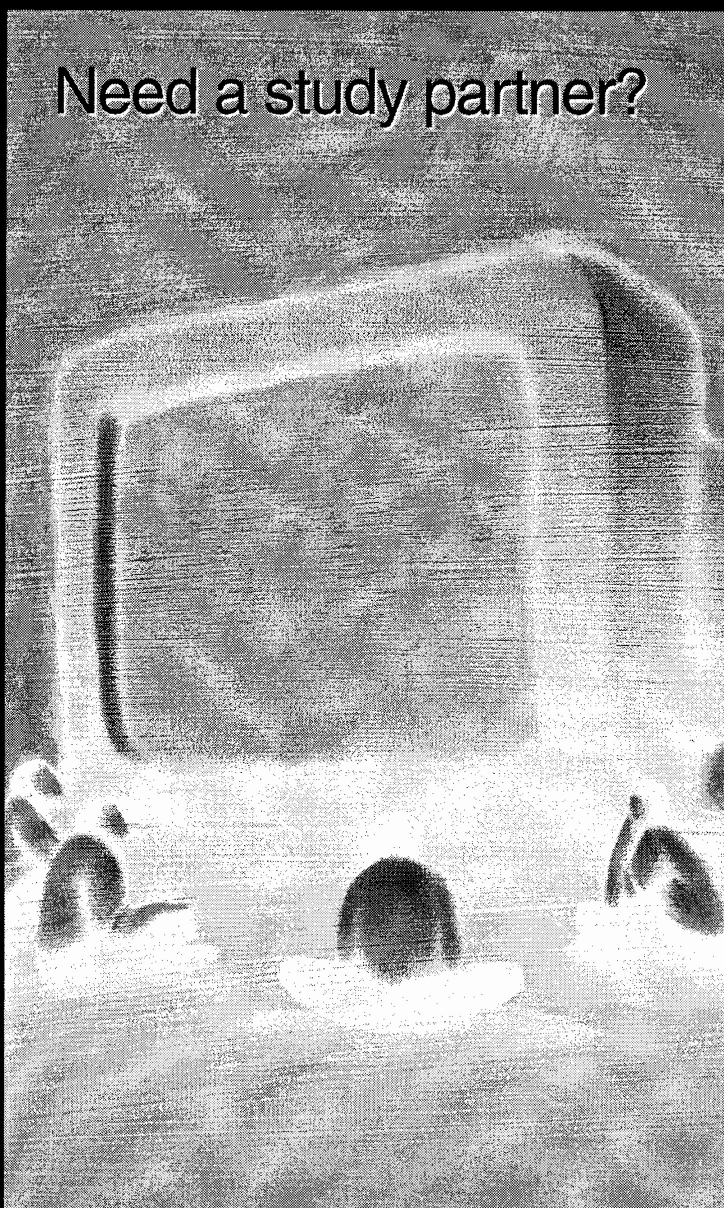

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