Technical Note: Simplified Equation for Simulating Velocity-Sensitive Corrosion in the Rotating Cylinder Electrode at Higher Reynolds Numbers

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ABSTRACT

A straightforward, explicit equation was developed previously to estimate an "equivalent" velocity in the rotating cylinder electrode that might best simulate the velocity in a pipe. That equation used friction factor vs Reynolds number relationships valid to Reynolds numbers of ~10^5. A similarly straightforward, explicit equation has been derived here that extends the estimate to much higher Reynolds numbers. Use of more complicated alternative equations while being more exact have been more difficult to use, requiring iteration to calculate the velocity. The equation reported here enables a reasonable estimate of the cylinder velocity to be calculated directly from the pipe velocity at higher Reynolds numbers without the need for iteration.

KEY WORDS: corrosion prediction, friction factor, laboratory simulation, pipe, rotating cylinder electrode, shear stress, velocity sensitivity

INTRODUCTION

Simulation of velocity-sensitive corrosion in the laboratory in one-phase systems is important for both prediction and diagnosis. One tool that has been fairly successfully applied for such simulation is the rotating cylinder electrode.[1] Of ongoing interest is development of easily used equations that enable the corrosion practitioner to estimate rotation velocities to be used in the laboratory that more closely simulate the hydrodynamic conditions in the field across a broad range of Reynolds numbers in both geometries. Over the last 25 years, papers have appeared proposing equations that were said to enable velocities in the rotating cylinder electrode to be estimated from velocities in the pipe. The predictions afforded by a number of these equations were recently compared and recommendations were made as to which equations might be the best to use.[2] Equations derived by Silverman[3] and Nešić, et al.,[4] were shown to provide comparable predictions of the appropriate rotating cylinder electrode velocity. The comparison was made for Reynolds numbers up to ~10^5.

These equations are rather simple to use since the velocity in the rotating cylinder can be calculated explicitly from the pipe velocity without iteration. The derivation used the friction factor vs Reynolds number relationship for the rotating cylinder electrode as developed by Eisenberg, et al.[5] That equation was derived from data corresponding to the Reynolds number range from ~10^3 to ~10^5. This upper limit implies that the error in the proposed equations would increase as the Reynolds number of the rotating cylinder that corresponds to the actual pipe velocity increases above that value.

Darby, et al., proposed an equation to overcome this Reynolds number limitation.[6] They obtained a friction factor vs Reynolds number relationship by curve fitting the Prandtl equation for smooth pipes and the similar equation derived by Theodorsen and Regier[7] for the rotating cylinder. Each of these equations is valid to Reynolds numbers much larger than 10^5, possibly as high as 10^7 for the pipe and 10^6 for the rotating cylinder. One shortcoming of their pro-

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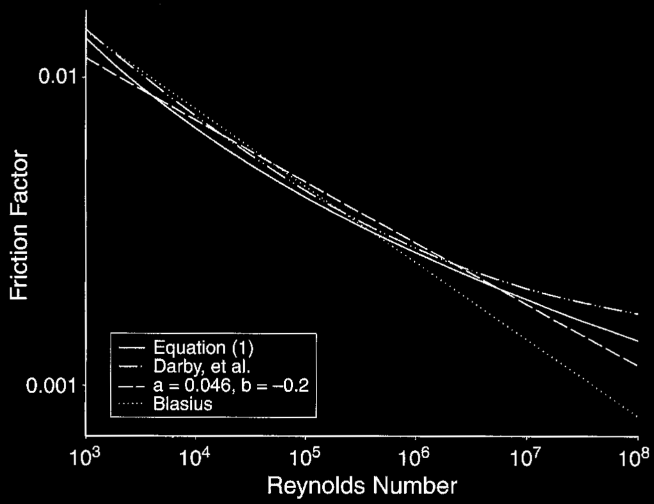

FIGURE 1. *Friction factor as a function of Reynolds number in a pipe.*

posed equation is that the velocity of the rotating cylinder electrode could not be calculated explicitly from the velocity of the pipe because the rotating cylinder electrode velocity occurred on both sides of the resulting equation. An iterative algorithm was required to solve the equation.

An alternative equation that provides a reasonable estimate of a simulating velocity at Reynolds numbers higher than those referenced earlier,[2] which enables the rotating cylinder electrode to be estimated explicitly from the pipe velocity without the need for iteration by computer, would be desirable for easier design of an experimental protocol. This paper proposes one such equation.

EQUATION DERIVATION

The approach was to start with the equations that fit the actual friction factor vs Reynolds number data for both the smooth pipe (Prandtl-von-Karman theory) and the smooth rotating cylinder electrode as presented by Theodorsen and Regier. For both geometries, the equation is of the form:

$$\frac{1}{\sqrt{f}} = C_i + C_j \mathrm{Log}_{10}(\mathrm{Re}\sqrt{f}) \qquad (1)$$

where C_i is ~0.4 for the pipe and –0.6 for the cylinder, and C_j is 4.0 for the pipe and 4.07 for the cylinder. Then, an equation of the form of:

$$f = a\,\mathrm{Re}^b \qquad (2)$$

was used to approximate Equation (1) for both geometries. The hope was that this simple equation would provide a practical alternative to Equation (1) across the range of Reynolds numbers for pipes from ~2×10^3 to ~10^7 and for the rotating cylinder from ~200 to >10^6. This simple form of the friction factor vs Reynolds number equation was chosen because its use for both geometries would ensure that the rotating cylinder velocity could be calculated explicitly from the pipe velocity. The issue is that in log(f) vs log(Re) coordinates, Equation (1) is curved and Equation (2) is a straight line.

The constants of 0.046 for a and –0.2 for b have been suggested as practical approximations to Equation (1) for Reynolds numbers from ~3×10^4 to >10^6.[8] A linear regression of Equation (2) was run vs the log(f) vs log(Re) relationship as calculated from Equation (1) for pipes with the constants provided by Theodorsen and Regier and assuming validity to a Reynolds number of 10^7. The regression resulted in values of a and b of 0.048 and –0.21, respectively, very close to the values mentioned above. The values of 0.046 and –0.2 were used in this derivation. The difference between this approach and that of Darby, et al., was their curve fitting of an equation of the form:

$$f = c + a\,\mathrm{Re}^b \qquad (3)$$

for both the rotating cylinder and the pipe. Inclusion of the additional constant, while being more accurate in capturing some of the curvature in the actual log(f) vs log(Re) relationship, led to the more complicated resulting expression requiring the iterative approach to relating velocities.

Figure 1 shows a plot of the friction factor vs Reynolds number for the four equations: Theodorsen and Regier (Prandtl-von Karman theory); Darby, et al.; Equation (2) with constants of 0.046 and –0.2; and the Blasius equation in which a is 0.0792 and b is –0.25. Equation (1) was solved iteratively by means of a FORTRAN program written to perform the calculation. As shown in Figure 2, the Blasius equation begins to deviate significantly in the range of Reynolds numbers greater than 10^5 to 5×10^5. The equation derived by Darby, et al., tracks the curvature of Equation (1) within ~5% to 10% up to a Reynolds number of ~10^7. Equation (2), with the constants of 0.046 and –0.2, while being a straight line, has only a marginally greater error in spots up to a Reynolds number of ~10^7.

For a smooth cylinder, Theodorsen and Regier[7] determined that Equation (1) provides a good fit to the friction factor vs Reynolds number data with constants as mentioned previously. They used this equation to fit data to Reynolds numbers up to ~10^6. This equation also forms a curved line when plotted on log-log coordinates of friction factor vs Reynolds number. Eisenberg, et al., derived a simpler form of the equation using results for mass transfer of ferricyanide/ferrocyanide and the Chilton-Colburn analogy.[5] This often-used equation was derived from results in the Reynolds number range from 10^3 to 10^5, where the curvature in the mass-transfer rate vs Reynolds number on log-log coordinates can be represented fairly well by a straight line. Darby, et al., again, used an equation of the form of Equation (3) to curve fit the fraction factor—Reynolds number relationship presented by Theodorsen and Regier. This equation, while again capturing some of the curvature of the plot and extending the accuracy to higher Reynolds numbers, increases the complexity of any equation that might be derived to relate the rotating cylinder electrode velocity to that in other geometries.

Once again, the simplified approach of assuming an equation of the form of Equation (2) across the Reynolds number range used by Theodorsen and Regier was examined to see if it can adequately capture enough of the actual friction factor vs Reynolds number relationship to be usable. As for the pipe, a linear regression of Equation (2) in logarithmic coordinates (log[f] vs log[Re]) was performed against data calculated from Equation (1) for the rotating cylinder. The regression was made to Reynolds numbers of 10^7. The resulting equation is:

$$f = 0.0977\,Re^{-0.257} \tag{4}$$

Figure 2 shows a plot of the friction factor vs Reynolds number for the four rotating cylinder equations (Theodorsen and Regier; Darby, et al.; Eisenberg, et al.; and Equation [4]). Equation (1) was solved iteratively by means of a FORTRAN program written to perform the calculation. As shown in Figure 2, the Eisenberg, et al., equation deviates significantly above Reynolds numbers of ~5×10^5. The equation derived by Darby, et al., tracks the curvature of Equation (1) within ~10% to 20% across the entire range of Reynolds numbers used. Equation (4), while being a straight line, has only a marginally greater error in lower and upper Reynolds number regions. But, only a relatively small amount of accuracy seems to be lost when using Equation (4) instead of Equation (3).

The revised equation that relates the rotating cylinder electrode velocity to the pipe velocity was derived following the procedure outlined previously.[3] The best relationship between the mass-transfer coefficient and friction factor was developed by eliminating the explicit dependence on Reynolds number using each of the above-mentioned relationships. Then, the mass-transfer coefficients were equated. The resulting equations were rearranged so that the functionality provides the relationship between velocities.

The modified equation was derived as follows:

For the rotating cylinder electrode, the mass-transfer coefficient can be related to the fluid shear stress by:

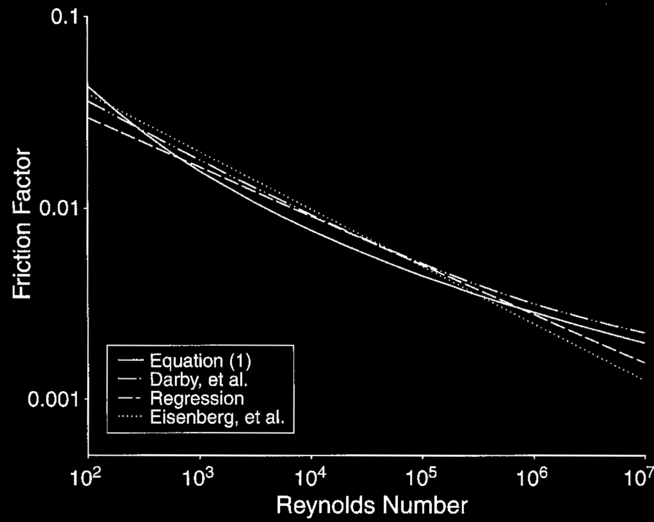

FIGURE 2. *Friction factor as a function of Reynolds number in a rotating cylinder.*

$$k_1 = \frac{\tau_w}{\rho u_1} Sc^{-0.644} = \frac{f}{2} u_1 SC^{-0.644} \tag{5}$$

Substituting Equation (4) into Equation (5) results in:

$$k_1 = 0.0489\,Re^{-0.257}\,u_1 Sc^{-0.644} \tag{6}$$

The exponent on the Schmidt number was assumed to be that given by Eisenberg, et al.[5]

Following Shaw and Hanratty,[9] the mass-transfer coefficient for the pipe can be related to the fluid shear stress and friction factor by:

$$k_2 = 0.0889\sqrt{\frac{\tau_w}{\rho}} Sc^{-0.704} = 0.0889\sqrt{\frac{f}{2}} u_2 Sc^{-0.704} \tag{7}$$

As mentioned previously, this expression has been verified for $4 \times 10^3 < Re < 10^5$ and $400 < Sc < 4 \times 10^4$. The assumption is being made that Equation (7) is valid for higher Reynolds numbers. Substituting Equation (2) with a = 0.046 and b = –0.2 into Equation (7) results in:

$$k_2 = 0.0135\,Re^{-0.1}\,u_2 Sc^{-0.704} \tag{8}$$

The velocity of the rotating cylinder that creates a mass-transfer coefficient equivalent to that in a pipe was obtained by setting Equation (6) equal to Equation (8) and by solving for the velocity of the cylinder in terms of the velocity in the pipe. The resulting equation is:

$$u_1 = 0.1768\left[\left(\frac{d_1^{0.346}}{d_2^{0.135}}\right)\left(\frac{\mu}{\rho}\right)^{-0.211} Sc^{-0.0808}\right] u_2^{1.211} \tag{9}$$

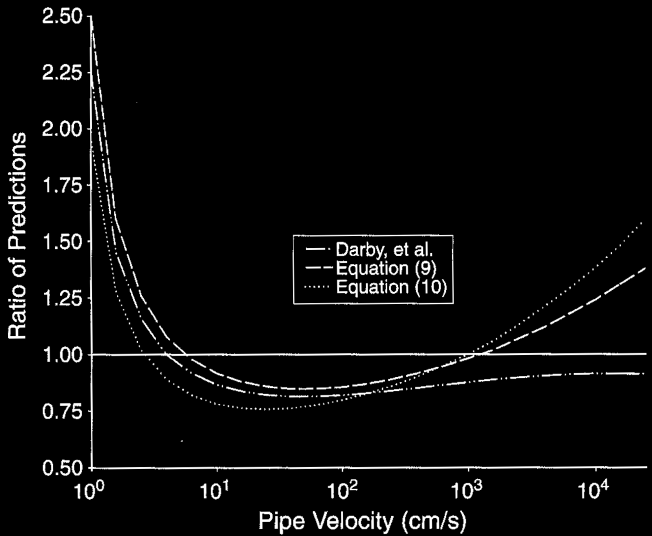

FIGURE 3. *Ratio of "equivalent" cylinder velocities to those estimated by using Equation (1) (pipe diameter = 5 cm, cylinder diameter = 2.54 cm).*

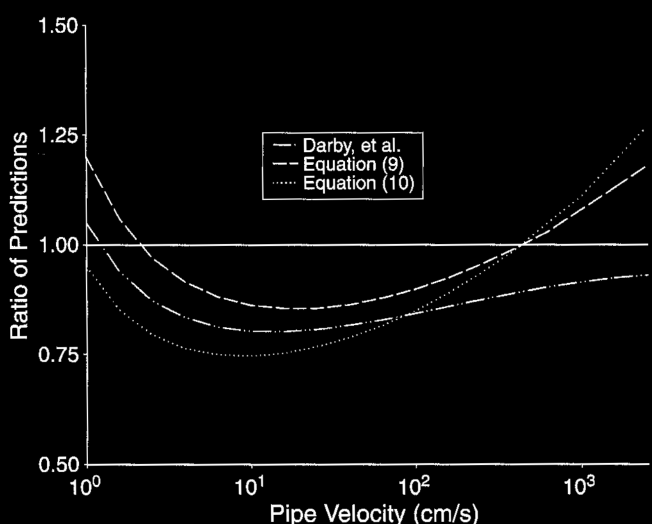

FIGURE 4. *Ratio of "equivalent" cylinder velocities to those estimated by using Equation (1) (pipe diameter = 50 cm, cylinder diameter = 10.16 cm).*

For reference, the previous equation derived using the Eisenberg, et al., friction factor vs Reynolds numbers relationship[3] is:

$$u_1 = 0.1184\left[\left[\left(\frac{d_1^{3/7}}{d_2^{5/28}}\right)\left(\frac{\mu}{\rho}\right)\right]^{-0.25} Sc^{-0.0857}\right]u_2^{1.25} \qquad (10)$$

Figures 3 and 4 show the ratios of each equivalent rotating cylinder electrode velocity as estimated from Equation (9), Equation (10), and from the equation developed by Darby, et al., (Equation [13] in Reference 6) to the equivalent velocity as estimated using the friction factors calculated from Equation (1) for the pipe and rotating cylinder geometries. That latter calculation was performed as follows. A pipe velocity was chosen and the corresponding friction factor obtained by iteration from Equation (1). Then, the equality of Equations (6) and (8) was used to obtain the cylinder velocity from the pipe velocity, again by iteration. A FORTRAN program was written to calculate all "equivalent" rotating cylinder velocities as a function of the pipe velocity. The Darby, et al., equation also had to be solved iteratively to obtain the "equivalent" rotating cylinder velocity from the pipe velocity. An algorithm was written to do so. Two cases are shown, for a pipe diameter of 5 cm and a cylinder diameter of 2.54 cm in Figure 3 and for a pipe diameter of 50 cm and a cylinder diameter of 10.16 cm in Figure 4. The upper pipe velocity in each case corresponds to a pipe Reynolds number of 10^7. The largest Reynolds number for the rotating cylinder is ~10^7 for the 2.54-cm-diameter case and slightly less for the 10.16-cm case. The line crossing at 1 would signify perfect agreement.

The cases were chosen to represent using a narrow cylinder to simulate a narrow pipe and a wide cylinder to simulate a wide pipe. The equation developed by Darby, et al., seems to provide a better estimate at pipe Reynolds numbers greater than ~10^6 (about a 10% error at a Reynolds number of 10^7 vs about a 25% error for Equation [10] in Figure 3, slightly less in Figure 4). But, the key point is that Equation (9), an equation that can be solved explicitly and is fairly easy to use, can still provide a reasonable prediction of the equivalent cylinder velocity from which a practical experimental evaluation can be designed. Equation (10) has increasing deviation from and produces a higher prediction than Equation (9). In both cases, the deviation from 1 becomes most marked at Reynolds numbers above ~10^6.

DISCUSSION

Equation (1) for both the pipe and cylinder are curve fit to data. The equation for the rotating cylinder was derived from data for Reynolds numbers between ~250 and ~5×10^5. Extrapolation of this equation to Reynolds numbers higher than ~10^6 must be made with caution. All of the equations are for the smooth cylinder. As discussed by others,[1,6] not only do the constants in Equation (2) change as surface roughness changes but when roughness is present the friction factor becomes an additional function of the surface roughness divided by the cylinder diameter and is usually written as this ratio raised to a power. Though equations such as (9) and (10) can be used as starting points from which to design an experimental program, the actual "equivalent" velocity could change as corrosion proceeds and the surface becomes rougher or as some thicker surface films introduce an additional diffusion limitation. For this reason, the differences shown in Figures 3 and 4 assume somewhat less significance when designing an experimental program. In fact, one reasonable approach would be to use Equations (9) or (10), the choice depending on whether one is above or below the Reynolds number limits for the underlying equations, to define a "center-point" velocity, and then design a laboratory protocol that brackets that cylinder velocity by velocities that span about an order of magnitude around it. In most instances, therefore, use of the explicit Equation (9) or possibly even Equation (10) would suffice.

One last cautionary note should be added about this entire approach of using one type of geometry to simulate another. The rotating cylinder electrode, or for that matter any geometry, should not be assumed to be the appropriate simulating geometry without first assuring oneself that the corrosion mechanism is the same as in the field geometry. Instances have been shown where the two mechanisms may be different, which could result in misleading predictions.[10]

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LIST OF SYMBOLS

C_i, C_j = constants in Equation (1)
a, b, c, = constants in Equations (2) and (3)
d = diameter
D = diffusion coefficient
f = friction factor (dimensionless)
Re = Reynolds number (dimensionless) $\left(\frac{\rho u d}{\mu}\right)$
Sc = Schmidt number (dimensionless) $\left(\frac{\mu}{\rho D}\right)$
u = velocity of fluid in pipe or cylinder
μ = absolute viscosity
ρ = fluid density
τ_w = fluid shear stress at the wall

SUBSCRIPTS

1 = cylinder
2 = pipe